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POLYNOMIAL QUANTIZTION AND OVERALGEBRA FOR HYPERBOLOID OF ONE SHEET

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Abstract. We show that the multiplication of symbols in polynomial quantization is exactly an action of an overalgebra on the space of these symbols
Keywords: quantization; representations; hyperboloids; Poisson transforms

In [1] we constructed quantization in the spirit of Berezin on para-Hermitian symmetric spaces G/H, see also [2]. In [3] we showed that this quantization, anyway polynomial quantization – the most algebraic variant of quantization, can be considered as a part of the representation theory. In present paper we continue our activity in this direction, namely, we show that the multiplication of symbols is exactly an action of an overalgebra on the space of symbols, see Theorem 2. Here we restrict ourselves to a hyperboloid of one sheet in $\mathbb{R}^3$. Besides, we write explicit formulae of this action.

The study of actions of overalgebras is a new theme, opened by Yu. A. Neretin and the author [4–6].

In this paper the group G is the group $\mathrm{SL}(2,\mathbb{R})$, the subgroup H consists of diagonal matrices, the space G/H is a hyperboloid of one sheet in $\mathbb{R}^3$. The overgroup $\widetilde{G} = G \times G$ contains three subgroups G^d, G_1 и G_2 isomorphic to G. Namely, they consist of pairs (g,g), (g,E), (E,g), respectively. Here E is identity matrix, $g \in G$.

Let $\mathfrak{g}$ be the Lie algebra of G. Then the Lie algebras of $\widetilde{G}$ and G^d, G_1, G_2 are $\widetilde{\mathfrak{g}} = \mathfrak{g} + \mathfrak{g}$ and $\mathfrak{g}^d$, $\mathfrak{g}_1$, $\mathfrak{g}_2$, respectively. In order to write an action of the overalgebra $\widetilde{\mathfrak{g}}$, it is sufficient to take some subspace complementary to $\mathfrak{g}^d$. Now we take the subalgebra $\mathfrak{g}_2$. It consists of pairs $(0,X)$, where $X \in \mathfrak{g}$.

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The group G consists of real matrices of the second order with unit determinant:

$$g = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}, \quad \alpha\delta - \beta\gamma = 1. \tag{1}$$

Changing in (1) $\alpha \leftrightarrow \delta$ and $\beta \leftrightarrow \gamma$, we obtain an involution $g \mapsto \widehat{g}$ in G given by

$$\widehat{g} = \begin{pmatrix} \delta & \gamma \\ \beta & \alpha \end{pmatrix}.$$

The Lie algebra $\mathfrak{g}$ of the group G consists of real matrices of the second order with zero trace. A basis in $\mathfrak{g}$ consists of matrices:

$$L_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad L_1 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}, \quad L_+ = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}. \tag{2}$$

The commutation relations are:

$$[L_+, L_-] = -2L_1\,, \quad [L_+, L_1] = -L_+\,, \quad [L_1, L_-] = -L_-\,. \tag{3}$$

Denote by $\mathrm{Env}\,(\mathfrak{g})$ the universal enveloping algebra of the Lie algebra $\mathfrak{g}$.

Recall some material on representations of G. We shall use the notation

$$t^{\lambda,\nu} = |t|^\lambda \mathrm{sgn}^\nu t\,, \quad t \in \mathbb{R}^* = \mathbb{R} \setminus \{0\},\ \lambda \in \mathbb{C},\ \nu = 0, 1.$$

For $\sigma \in \mathbb{C}$, $\nu = 0, 1$, let us denote by $\mathcal{D}_{\sigma,\nu}(\mathbb{R})$ the space of functions f in $C^\infty(\mathbb{R})$ such that the function $\widehat{f}(t) = t^{2\sigma,\nu} f(1/t)$ belongs to $C^\infty(\mathbb{R})$ too. The representation $\pi_{\sigma,\nu}$ of the group G acts on $\mathcal{D}_{\sigma,\nu}(\mathbb{R})$ by (we consider that G acts from the right):

$$(\pi_{\sigma,\nu}(g) f)\,(t) = f\,(t \cdot g)\,\,(\beta t + \delta)^{2\sigma,\nu}, \quad t \cdot g = \frac{\alpha t + \gamma}{\beta t + \delta}\,.$$

The *contragredient* representation $\widehat{\pi}_{\sigma,\nu}$ is defined by the involution $g \mapsto \widehat{g}$, so that

$$(\widehat{\pi}_{\sigma,\nu}(g) f)\,(t) = f\,(t \cdot \widehat{g})\,\,(\gamma t + \alpha)^{2\sigma,\nu}.$$

Representations $\pi_{\sigma,\nu}$ and $\widehat{\pi}_{\sigma,\nu}$ are equivalent by means of the operator $f \mapsto \widehat{f}$.

Any irreducible finite-dimensional representation ρ_k of the group G is labelled by the number k (the *highest weight*) such that $2k \in \mathbb{N} = \{0, 1, 2, \ldots\}$. It acts on the space V_k of polynomials $\varphi(t)$ in t of degree $\leqslant 2k$ (so that $\dim V_k = 2k + 1$) by

$$(\rho_k(g)\,\varphi)\,(t) = \varphi\,(t \cdot g)\,\,(\beta t + \delta)^{2k}\,.$$

Operators corresponding to elements of $\mathfrak{g}$ and $\mathrm{Env}\,(\mathfrak{g})$ in representations $\pi_{\sigma,\nu}$ do not depend on ν, so we do not write ν in indexes.

For basis elements (2) we have

$$\pi_\sigma(L_-) = \frac{d}{dt}, \quad \pi_\sigma(L_1) = t\,\frac{d}{dt} - \sigma\,, \quad \pi_\sigma(L_+) = t^2\,\frac{d}{dt} - 2\sigma t.$$

and $\widehat{\pi}_\sigma(L_\pm) = -\pi_\sigma(L_\mp)$, $\widehat{\pi}_\sigma(L_1) = -\pi_\sigma(L_1)$. Replacing here σ by k, we obtain formulas for ρ_k.

On the product $\varphi\psi$ of functions φ and ψ the differential operators $\pi_\sigma(L)$, $L \in \mathfrak{g}$, (they have the first order) act as follows:

$$\pi_\sigma(L)(\varphi\psi) = (\pi_\sigma(L)\varphi)\cdot\psi + \varphi\cdot(\pi_0(L)\psi)\,, \tag{4}$$

and similarly to $\widehat{\pi}_\sigma$.

An operator $A_{\sigma,\nu}$ defined by

$$(A_{\sigma,\nu}f)(t) = \int_{-\infty}^{\infty} (1-ts)^{-2\sigma-2,\,\nu} f(s)\,ds$$

intertwines $\pi_{\sigma,\nu}$ and $\widehat{\pi}_{-\sigma-1,\nu}$:

$$\widehat{\pi}_{-\sigma-1,\nu}(g)A_{\sigma,\nu} = A_{\sigma,\nu}\pi_{\sigma,\nu}(g)\,,$$

and also $\widehat{\pi}_{\sigma,\nu}$ and $\pi_{-\sigma-1,\nu}$. The composition $A_{\sigma,\nu}$ and $A_{-\sigma-1,\nu}$ is a scalar operator:

$$A_{-\sigma-1,\nu}\,A_{\sigma,\nu} = \frac{1}{c(\sigma,\nu)}\cdot E,$$

where

$$c(\sigma,\varepsilon) = \frac{2\sigma+1}{2\pi}\cdot\frac{(-1)^\nu + \cos 2\sigma\pi}{\sin 2\sigma\pi}\,.$$

Let us realize the space $\mathbb{R}^4$ of vectors $x = (x_0, x_1, x_2, x_3)$ as the space of real 2×2 matrices:

$$x = \frac{1}{2}\begin{pmatrix} x_0 - x_3 & -x_1 + x_2 \\ x_1 + x_2 & x_0 + x_3 \end{pmatrix}.$$

The overgroup $\widetilde{G}$ acts as follows:

$$x \mapsto g_1^{-1}xg_2, \quad (g_1, g_2) \in \widetilde{G}.$$

Let $\mathcal{C}$ be the cone $\det x = 0$, $x \neq 0$. For $\sigma \in \mathbb{C}$, $\nu = 0, 1$, let $\mathcal{D}_{\sigma,\nu}(\mathcal{C})$ denote the space of C^∞ functions f on the cone $\mathcal{C}$ homogeneous of degree 2σ and parity ν:

$$f(tx) = t^{2\sigma,\nu}f(x), \quad t \in \mathbb{R}^*.$$

Let $\widetilde{R}_{\sigma,\nu}$ be the representation of $\widetilde{G}$ by translations on the space $\mathcal{D}_{\sigma,\nu}(\mathcal{C})$ (in fact, it is a representation of the group $\mathrm{SO}_0(2,2)$ associated with a cone, $\widetilde{G}$ covers $\mathrm{SO}_0(2,2)$ with multiplicity 2):

$$(\widetilde{R}_{\sigma,\nu}(g_1,g_2)f)(x) = f(g_1^{-1}xg_2).$$

The section $\mathcal{X}$ of $\mathcal{C}$ by plane $(\operatorname{tr} x) = 1$ can be identified with a hyperboloid of one sheet $-x_1^2 + x_2^2 + x_3^2 = 1$ in $\mathbb{R}^3$. Restrictions of functions in $\mathcal{D}_{\sigma,\nu}(\mathcal{C})$ to $\mathcal{X}$ form a space $\mathcal{D}_{\sigma,\nu}(\mathcal{X})$

of functions on $\mathcal{X}$. It is contained in $C^\infty(\mathcal{X})$ and contains $\mathcal{D}(\mathcal{X})$. In the realization on $\mathcal{X}$ the representation $\widetilde{R}_{\sigma,\nu}$ is:

$$(R_{\sigma,\nu}(g_1,g_2)f)(x) = f\left(\frac{g_1^{-1}xg_2}{\mathrm{tr}(g_1^{-1}xg_2)}\right)\left\{\mathrm{tr}(g_1^{-1}xg_2)\right\}^{2\sigma,\nu}, \quad x \in \mathcal{X}.$$

The section $\mathcal{X}$ is invariant with respect to the action $x \mapsto g^{-1}xg$ of $G^d = G$, it is just the space G/H. The restriction of $\widetilde{R}_{\sigma,\nu}$ to $G^d = G$ is the quasiregular representation U of G on $\mathcal{X}$. It preserves the space $S(\mathcal{X})$ of polynomials on $\mathcal{X}$ and decomposes in the direct sum: $U = \rho_0 + \rho_1 + \rho_2 + \ldots$ with the corresponding decomposition $S(\mathcal{X}) = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2 + \ldots$

Introduce on $\mathcal{X}$ horospherical coordinates ξ, η:

$$x = \frac{1}{N}\left(\begin{array}{cc} -\eta\xi & -\eta \\ \xi & 1 \end{array}\right), \quad N = N(\xi,\eta) = 1 - \xi\eta,$$

so that

$$x_1 = \frac{\xi+\eta}{N}, \; x_2 = \frac{\xi-\eta}{N}, \; x_3 = \frac{1+\xi\eta}{N}.$$

In these coordinates, to basis elements (2) the following operators correspond:

$$\widetilde{R}_\sigma(0, L_-) = \frac{\partial}{\partial\xi} - 2\sigma\,\frac{\eta}{N},$$
$$\widetilde{R}_\sigma(0, L_1) = \xi\frac{\partial}{\partial\xi} - \sigma\,\frac{\xi\eta+1}{N},$$
$$\widetilde{R}_\sigma(0, L_+) = \xi^2\frac{\partial}{\partial\xi} - 2\sigma\,\frac{\xi}{N}.$$

Recall some material on *polynomial quantization.* As a supercomplete system we take the kernel of the intertwining operator $A_{-\sigma-1,\nu}$, namely,

$$\Phi_{\sigma,\nu}(\xi,\eta) = N(\xi,\eta)^{2\sigma,\nu}.$$

This function has an invariance property

$$\Big[\pi_\sigma(g) \otimes \widehat{\pi}_\sigma(g)\Big]\Phi_{\sigma,\nu}(\xi,\eta) = \Phi_{\sigma,\nu}(\xi,\eta).$$

This formula can be rewritten as

$$\Big(\pi_\sigma(g^{-1}) \otimes 1\Big)\,\Phi_{\sigma,\nu}(\xi,\eta) = \Big(1 \otimes \widehat{\pi}_\sigma(g)\Big)\,\Phi_{\sigma,\nu}(\xi,\eta). \tag{5}$$

For elements L of the Lie algebra $\mathfrak{g}$, formula (5) gives:

$$-\Big(\pi_\sigma(L) \otimes 1\Big)\,\Phi_{\sigma,\nu}(\xi,\eta) = \Big(1 \otimes \widehat{\pi}_\sigma(L)\Big)\,\Phi_{\sigma,\nu}(\xi,\eta). \tag{6}$$

Covariant symbols of operators $\pi_\sigma(X)$, $X \in \mathrm{Env}\,(\mathfrak{g})$, are functions $F(x)$ on $\mathcal{X}$ defined as follows:

$$F(\xi,\eta) = \frac{1}{\Phi_{\sigma,\nu}(\xi,\eta)}\Big(\pi_\sigma(X)\otimes 1)\Phi_{\sigma,\nu}(\xi,\eta\Big).$$

In particular, covariant symbols for basis elements (2) are multiplied by $(-\sigma)$ polynomials $x_1 - x_2$, x_3, $x_1 + x_2$, respectively.

The multiplication of operators gives rise to the multiplication of covariant symbols, denote it by $*$. Namely, let F_1 and F_2 be covariant symbols of operators D_1 and D_2 respectively. Then the covariant symbol $F_1 * F_2$ of the product $D_1 D_2$ is

$$(F_1 * F_2)(\xi,\eta) = \frac{1}{\Phi_{\sigma,\nu}(\xi,\eta)}\,(D_1\otimes 1)\Big(\Phi_{\sigma,\nu}(\xi,\eta)\,F_2(\xi,\eta)\Big)\,. \tag{7}$$

Let V be a covariant symbol of the *first order* (corresponding to an element L of the Lie algebra $\mathfrak{g}$). Let F be an *arbitrary* covariant symbol (corresponding to an element X of the universal enveloping algebra $\mathrm{Env}\,(\mathfrak{g})$)

Theorem 1. *We have (the point means pointwise multiplication)*

$$V * F = V\cdot F + (\pi_0(L)\otimes 1))\,F \tag{8}$$

$$F * V = V\cdot F - (1\otimes \widehat{\pi}_0(L))\,F \tag{9}$$

Proof. To prove (8), we take in formula (7) $D_1 = \pi_\sigma(L)$ and $F_2 = F$, then we differentiate by (4), as a result we get (8).

Now let $D = \pi_\sigma(X)$. Since $(D\pi_\sigma(L))\otimes 1 = (D\otimes 1)(\pi_\sigma(L)\otimes 1)$, we have

$$F * V = \frac{1}{\Phi_{\sigma,\nu}}\,(D\otimes 1)(\pi_\sigma(L)\otimes 1)\Phi_{\sigma,\nu},$$

then by (6) we can change here the latter operator by the operator $\{-(1\otimes\widehat{\pi}_\sigma(L))\}$ and then transpose it with $D\otimes 1$ since they act on different variables. We obtain

$$F * V = -\frac{1}{\Phi_{\sigma,\nu}}\Big(1\otimes\widehat{\pi}_\sigma(L)\Big)(\Phi_{\sigma,\nu}F), \tag{10}$$

then we differentiate by (4) and use (6) again. It gives (9). □

Theorem 2. *The multiplication of covariant symbols F by first order symbols V is the action of the overalgebra $\widetilde{\mathfrak{g}}$ on the space of covariant symbols:*

$$V * F = \widetilde{R}_\sigma(0,L)F, \quad F * V = -\widetilde{R}_\sigma(L,0)F. \tag{11}$$

Proof. Formula (7) with $D_1 = \pi_\sigma(L)$ and $F_2 = F$ gives exactly the first formula in (11). The second formula is just (10). □

For $k\in\mathbb{N}$, we define the Poisson kernel $P_k(x;t)$ as follows. Denote

$$B(x;t) = B(\xi,\eta;t) = \frac{(t-\xi)(1-\eta t)}{N}\,, \tag{12}$$

then

$$P_k(x;t) = B(x;t)^k. \tag{13}$$

This kernel is a fixed vector in the tensor product $U \otimes \rho_k$:

$$(U(g) \otimes \rho_k(g) P_k)(x;t) = P_k(x;t), \quad g \in G.$$

Therefore, $P_k(x;t)$ is a generating function for polynomials in $\mathcal{H}_k$.

Let us introduce the following differential operators $S_k(X)$, $k \in \mathbb{N}$, and $E(X)$ in variable t, linearly depending on $X \in \mathfrak{g}$, for basic elements (2) they are

$$\begin{aligned}
&E(L_-) = 1, \quad S_k(L_-) = \frac{d^2}{dt^2},\\
&E(L_1) = t, \quad S_k(L_1) = t\frac{d^2}{dt^2} - (2k+1)\frac{d}{dt},\\
&E(L_+) = t^2, \quad S_k(L_+) = t^2\frac{d^2}{dt^2} - 2(2k+1)t\frac{d}{dt} + (2k+1)(2k+2).
\end{aligned}$$

The following commutation relations hold

$$\begin{aligned}
S_k([X,Y]) &= \rho_k(X)\, S_k(Y) - S_k(Y)\, \rho_{k+1}(X),\\
E([X,Y]) &= \rho_k(X)\, E(Y) - E(Y)\, \rho_{k-1}(X).
\end{aligned}$$

Then, let us introduce the following coefficients α_k, β_k, γ_k:

$$\begin{aligned}
\alpha_k &= \frac{2\sigma - k}{(2k+2)(2k+1)},\\
\beta_k &= -\frac{1}{2},\\
\gamma_k &= -\frac{(2\sigma + k + 1)k}{2(2k+1)}.
\end{aligned}$$

Theorem 3. *Let $X \in \mathfrak{g}$. The operator $\widetilde{R}_\sigma(0,X)$ acts on the Poisson kernel $P_k(x;t)$ as follows:*

$$\widetilde{R}_\sigma(0,X)\, P_k = \alpha_k \cdot S_k(X)\, P_{k+1} + \beta_k \cdot \rho_k(X)\, P_k + \gamma_k \cdot E(X)\, P_{k-1}, \tag{14}$$

in the left hand side the operator acts on a function of ξ, η, and in the left hand side the operators act on functions of t.

Proof. First we take $X = L_-$. Keeping in mind (12), (13), we find:

$$\left(\frac{\partial}{\partial \xi} - 2\sigma\, \frac{\eta}{N}\right) B^k = -kB^{k-1} \cdot \frac{1 - \eta t}{N} + (-2\sigma + k) B^k \cdot \frac{\eta}{N}. \tag{15}$$

On the other hand, we compute $(\partial/\partial t)B^k$ and $(\partial^2/\partial t^2)B^{k+1}$:

$$\begin{aligned} \frac{\partial}{\partial t}B^k &= kB^{k-1}\cdot\frac{-2\eta t+\xi\eta+1}{N} \\ &= -kB^{k-1}+2kB^{k-1}\cdot\frac{1-\eta t}{N}, \qquad (16) \\ \frac{\partial^2}{\partial t^2}B^{k+1} &= (k+1)\left\{kB^{k-1}\cdot\frac{(-2\eta t+\xi\eta+1)^2}{N^2}-B^k\cdot\frac{2\eta}{N}\right\} \\ &= (k+1)\left\{kB^{k-1}-2(2k+1)B^k\cdot\frac{\eta}{N}\right\}. \qquad (17) \end{aligned}$$

Expressing from (16) and (17) the second summands in right hand sides, substituting in (15), we obtain (14) for $X = L_-$. Now for $X = L_1$ and $X = L_+$, we use equality (14) with $X = L_-$ already proved and commutation relations – successively the first and the second ones in (3), and corresponding commutation relations for operators $S_k(X)$ and $E(X)$. $\square$

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ПОЛИНОМИАЛЬНОЕ КВАНТОВАНИЕ И НАДАЛГЕБРА ДЛЯ ОДНОПОЛОСТНОГО ГИПЕРБОЛОИДА

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Аннотация. Мы показываем, что умножение символов в полиномиальном квантовании есть в точности действие надалгебры на пространстве этих символов.
Ключевые слова: квантование; представления; гиперболоиды; преобразования Пуассона

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